

Recovering information of tunneling spectrum from Weakly Isolated Horizon

Ge-Rui Chen* and Yong-Chang Huang

Institute of Theoretical Physics, Beijing University of Technology, Beijing, 100124, China

Abstract

In this paper we investigate the properties of tunneling spectrum from weakly isolated horizon(WIH). We find that there are correlations among Hawking radiations from weakly isolated horizon, the information can be carried out in terms of correlations between sequential emissions, and the radiation is an entropy conservation process. We generalize Refs.[11–13]’ results to a more general spacetime. Through revisiting the calculation of tunneling spectrum of weakly isolated horizon, we find that Ref.[12]’s requirement that radiating particles have the same angular momenta of unit mass as that of black hole is not needed, and the energy and angular momenta of emitting particles are very arbitrary which should be restricted only by keeping the cosmic censorship of black hole.

Keywords: weakly isolated horizon, tunneling spectrum, correlation, mutual information, entropy conservation process

*Electronic address: chengerui@emails.bjut.edu.cn

I. INTRODUCTION

In the 1970s, Hawking's astounding discovery that black holes radiate black body spectrum[1, 2] had greatly stimulated the development of the theory of black hole, and then four laws of black hole thermodynamics were established[3, 4]. Hawking radiation gives us new insights into gravity physics and also provides some hints of quantum gravity. From Hawking's famous work, people know that black holes are not the final state of stars, and, with the emission of Hawking radiation, they could lose energy, shrink, and eventually evaporate completely. However, because of the quality of purely thermal spectrum, it also sets up a disturbing and difficult problem: what happens to information during black hole evaporation? This scenario is inconsistent with the unitary principle of quantum mechanics[5–8]. About the year of 2000, Parikh and Wilczek, contemplating Hawking's heuristic picture of tunneling triggered by vacuum fluctuations near the horizon, proposed a semiclassical method to investigate the emission rate by treating Hawking radiation as a tunneling process [9, 10]. This method considers the back reaction of the emission particle to the spacetime, and does not fix the background spacetime. They found that the barrier of tunneling is created by the outgoing particle itself, and when energy conservation is considered, a non-thermal spectrum is given, which supports the underlying unitary theory.

In 2009, Refs.[11–13] gave more detail discussions about Parikh and Wilczek's non-thermal spectrum. They found that there are correlations among sequential Hawking radiations, the correlations equal to mutual information, and black hole radiation is an entropy conservation process, which is consistent with unitarity of quantum mechanics. Their discussions are based on stationary black holes, and we study this problem for weak isolated horizon[15–18]—a quasi-local defined black hole, and prove that for this kind of dynamical black holes, the information is also not lost, and is encoded into correlations between Hawking radiations. In our analysis Ref.[12]'s requirement that radiating particles have the same angular momenta of unit mass as that of black hole is not needed, and the energies and angular momenta of emitting particles are very arbitrary which should be restricted only by keeping the cosmic censorship of black hole.

This paper is organized as follows. In Section 2, we review the tunneling method to get the non-thermal spectrum of weakly isolated horizon. In Section 3, we investigate the qualities of this non-thermal spectrum. In the last Section, we give some discussions and

conclusions.

II. REVIEW PARIKH AND WILCZEK'S TUNNELING SPECTRUM FOR WEAKLY ISOLATED HORIZON

In this section we review the calculation of tunneling spectrum of WIH. We have some difference from the original discussion[19], and strictly follow Parikh and Wilczek's calculation[9, 10] which does not use explicitly the first law of black hole thermodynamics.

Ref.[17] established the first law of weakly isolated horizon thermodynamics,

$$\delta E = \frac{1}{8\pi}\kappa\delta A + \Omega\delta J = \frac{1}{2\pi}\kappa\delta S + \Omega\delta J. \quad (1)$$

The expressions of the surface gravity, angular velocity and horizon energy of weakly isolated horizon are given by

$$\kappa = \frac{R^4 - 4J^2}{2R^3\sqrt{R^4 + 4J^2}}, \Omega = \frac{2J}{R\sqrt{R^4 + 4J^2}}, E = \frac{\sqrt{R^4 + 4J^2}}{2R}, \quad (2)$$

where R is the horizon radius and is defined as

$$R \equiv \sqrt{\frac{A}{4\pi}}. \quad (3)$$

A is the area of any cross section of the horizon, so the entropy can also be expressed as

$$S = \frac{A}{4} = \pi R^2. \quad (4)$$

In the semiclassical limit, we can apply the WKB formula. The emission rate Γ can be given as

$$\Gamma \sim \exp(-2\text{Im } I), \quad (5)$$

where I is the action of the emitting particle.

The imaginary part of the action for a s-wave outgoing positive energy particle, from r_{in} to r_{out} , can be given as

$$\text{Im } I = \text{Im} \int_{r_{in}}^{r_{out}} p_r dr = \text{Im} \int_{r_{in}}^{r_{out}} \int_0^{p_r} dp_r dr. \quad (6)$$

From Hamilton's equation of the emitting particle,

$$dp_r = \frac{d\varepsilon}{\dot{r}} \quad (7)$$

where ε is the energy of the emitting particles, we can get

$$Im I = Im \int_{r_{in}}^{r_{out}} \int_0^\omega \frac{d\varepsilon}{\dot{r}} dr. \quad (8)$$

From Ref.[19], the outgoing geodesic is

$$\dot{r} = B_t(\varepsilon + \bar{\varepsilon})r + O(r^2) = \kappa r + O(r^2), \quad (9)$$

where $\kappa = B_t(\varepsilon + \bar{\varepsilon})$ is the surface gravity of the horizon, and is constant on the horizon.

So the imaginary part of action is

$$\begin{aligned} Im I &= Im \int_{r_{in}}^{r_{out}} \int_0^\omega \frac{d\varepsilon}{\kappa r + O(r^2)} dr = Im \int_0^\omega \int_{r_{in}}^{r_{out}} \frac{dr}{\kappa r + O(r^2)} d\varepsilon \\ &= Im \int_0^\omega [\pi i \frac{1}{\kappa}] d\varepsilon = \pi \int_0^\omega \frac{d\varepsilon}{\kappa}, \end{aligned} \quad (10)$$

where the integral of r is done by deforming the contour around the pole in the third equality.

For non-rotating WIH, $\Omega = 0, J = 0$, so we can get from (2)

$$\kappa = \frac{1}{4E}. \quad (11)$$

We fix the total mass of the space-time, and allow the black hole mass to fluctuate. After emitting a particle with energy ε the black hole mass becomes $E - \varepsilon$, so we obtain

$$Im I = \pi \int_0^\omega \frac{d\varepsilon}{\kappa'} = \pi \int_0^\omega 4E' d\varepsilon = \pi \int_0^\omega 4(E - \varepsilon) d\varepsilon = 4\pi\varepsilon(E - \frac{\varepsilon}{2}). \quad (12)$$

According to the definition of entropy of WIH,

$$S = \pi R^2 = 4\pi E^2, \quad (13)$$

we have the change of the entropy after the particle radiates,

$$\Delta S = 4\pi[(E - \varepsilon)^2 - E^2] = -8\pi\varepsilon(E - \frac{\varepsilon}{2}). \quad (14)$$

So we get the tunneling rate

$$\Gamma = \exp(-2Im I) = \exp[-8\pi\varepsilon(E - \frac{\varepsilon}{2})] = \exp(\Delta S) = \exp(4\pi[(E - \varepsilon)^2 - E^2]). \quad (15)$$

Next, we discuss the rotating WIH. From Eqs.(2), after some calculation, we can get

$$\begin{aligned} \Omega &= \frac{J}{2E(E^2 + \sqrt{E^4 - J^2})}, \quad \kappa = \frac{\sqrt{E^4 - J^2}}{2E(\sqrt{E^4 - J^2} + E^2)}, \\ S &= \pi R^2 = 2\pi(E^2 + \sqrt{E^4 - J^2}). \end{aligned} \quad (16)$$

For axial symmetric WIH, using the formula[19, 20], the action $Im I$ should be

$$\begin{aligned}
Im I &= Im \int [p_r dr - p_\phi d\phi] = Im \int [p_r - \frac{p_\phi \dot{\phi}}{\dot{r}}] dr = Im \int \int \frac{dH - \dot{\phi} dp_\phi}{\dot{r}} dr \\
&= Im \int \int \frac{d\varepsilon - \Omega dj}{\dot{r}} dr = Im \int \int \frac{dr}{\dot{r}} (d\varepsilon - \Omega dj) \\
&= Im \int \frac{\pi i}{\kappa} (d\varepsilon - \Omega dj) = \pi \int \frac{d\varepsilon - \Omega dj}{\kappa},
\end{aligned} \tag{17}$$

where we consider the s-wave, the particles radiate along the normal direction of the horizon, so $\dot{\phi} = \Omega$ according to the relationship $t^a \hat{=} B l^a - \Omega \phi^a$ [17, 19]. This is the acquirement for the emitting particles, and emitting particles do not need to have the original angular momentum of unit mass of black hole(see Ref.[12]).

When particle's self-gravitation is taken into account we should replace E and J with $E - \varepsilon$, and $J - j$, and substitute into the expression of κ and Ω in the last Eq.(17), so we get

$$\begin{aligned}
Im I &= \pi \int \frac{d\varepsilon - \frac{J-j}{2(E-\varepsilon)[(E-\varepsilon)^2 + \sqrt{(E-\varepsilon)^4 - (J-j)^2}]} dj}{\frac{\sqrt{(E-\varepsilon)^4 - (J-j)^2}}{2(E-\varepsilon)[\sqrt{(E-\varepsilon)^4 - (J-j)^2} + (E-\varepsilon)^2]}} \\
&= \pi \int \frac{2(E-\varepsilon)[\sqrt{(E-\varepsilon)^4 - (J-\varepsilon)^2} + (E-\varepsilon)^2]}{\sqrt{(E-\varepsilon)^4 - (J-\varepsilon)^2}} d\varepsilon \\
&\quad - \frac{J-j}{\sqrt{(E-\varepsilon)^4 - (J-\varepsilon)^2}} dj.
\end{aligned} \tag{18}$$

We do not need to do the integration directly. The change of back hole entropy after emitting a particle is

$$\Delta S = 2\pi[(E-\varepsilon)^2 + \sqrt{(E-\varepsilon)^4 - (J-j)^2}] - 2\pi[E^2 + \sqrt{E^4 - J^2}]. \tag{19}$$

We can get

$$\begin{aligned}
\frac{\partial(\Delta S)}{\partial \varepsilon} &= -4\pi[\frac{(E-\varepsilon)^3}{\sqrt{(E-\varepsilon)^4 - (J-j)^2}} + (E-\varepsilon)], \\
\frac{\partial(\Delta S)}{\partial j} &= 2\pi \frac{J-j}{\sqrt{(E-\varepsilon)^4 - (J-j)^2}},
\end{aligned} \tag{20}$$

and substitute the results into Eq(18), then we get

$$\begin{aligned}
Im S &= -\frac{1}{2}[\int \frac{\partial(\Delta S)}{\partial \varepsilon} d\varepsilon + \frac{\partial(\Delta S)}{\partial j} dj] \\
&= -\frac{1}{2}\Delta S.
\end{aligned} \tag{21}$$

So the tunneling rate is

$$\begin{aligned}\Gamma &= \exp(-2ImS) = \exp(\Delta S) \\ &= \exp(2\pi[(E - \varepsilon)^2 + \sqrt{(E - \varepsilon)^4 - (J - j)^2}] - 2\pi[E^2 + \sqrt{E^4 - J^2}]),\end{aligned}\quad (22)$$

and our next discussion is based on this equation.

III. INFORMATION RECOVERY FROM TUNNELING SPECTRUM OF WEAKLY ISOLATED HORIZON

In this section, we investigate the properties of tunneling spectrum from weakly isolated horizon following Refs.[11–13]. The probability for the emission of a particle with an energy ε_1 and an angular momentum j_1 is

$$\Gamma(\varepsilon_1, j_1) = \exp(2\pi[(E - \varepsilon_1)^2 + \sqrt{(E - \varepsilon_1)^4 - (J - j_1)^2}] - 2\pi[E^2 + \sqrt{E^4 - J^2}]). \quad (23)$$

And the probability for the emission of a particle with an energy ε_2 and an angular momentum j_2 is

$$\Gamma(\varepsilon_2, j_2) = \exp(2\pi[(E - \varepsilon_2)^2 + \sqrt{(E - \varepsilon_2)^4 - (J - j_2)^2}] - 2\pi[E^2 + \sqrt{E^4 - J^2}]). \quad (24)$$

Please note that ε_1, j_1 and ε_2, j_2 represent two different emitting particles, so the expressions should have the same form.

Let us consider a process. Firstly a particle with energy and angular momentum ε_1, j_1 emits, and then a particle with energy and angular momentum ε_2, j_2 radiates, so the probability for the emission of second particle is

$$\begin{aligned}\Gamma(\varepsilon_2, j_2 | \varepsilon_1, j_1) &= \exp(2\pi[(E - \varepsilon_1 - \varepsilon_2)^2 + \sqrt{(E - \varepsilon_1 - \varepsilon_2)^4 - (J - j_1 - j_2)^2}] \\ &\quad - 2\pi[(E - \varepsilon_1)^2 + \sqrt{(E - \varepsilon_1)^4 - (J - j_1)^2}]).\end{aligned}\quad (25)$$

which is the conditional probability and is different from the independent probability(24).

The emitting probability for two emissions with energies and angular momenta ε_1, j_1 and

ε_2, j_2 successively, can be deduced as follows

$$\begin{aligned}
\Gamma(\varepsilon_1, j_1, \varepsilon_2, j_2) &\equiv \Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) \\
&= \exp(2\pi[(E - \varepsilon_1)^2 + \sqrt{(E - \varepsilon_1)^4 - (J - j_1)^2}] - 2\pi[E^2 + \sqrt{E^4 - J^2}]) \\
&\quad \times \exp(2\pi[(E - \varepsilon_1 - \varepsilon_2)^2 + \sqrt{(E - \varepsilon_1 - \varepsilon_2)^4 - (J - j_1 - j_2)^2}] \\
&\quad - 2\pi[(E - \varepsilon_1)^2 + \sqrt{(E - \varepsilon_1)^4 - (J - j_1)^2}]) \\
&= \exp(2\pi[(E - \varepsilon_1 - \varepsilon_2)^2 + \sqrt{(E - \varepsilon_1 - \varepsilon_2)^4 - (J - j_1 - j_2)^2}] \\
&\quad - 2\pi[E^2 + \sqrt{E^4 - J^2}])
\end{aligned} \tag{26}$$

The last equality is nothing but $\Gamma(\varepsilon_1 + \varepsilon_2, j_1 + j_2)$, so we get

$$\Gamma(\varepsilon_1, j_1, \varepsilon_2, j_2) \equiv \Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) = \Gamma(\varepsilon_1 + \varepsilon_2, j_1 + j_2). \tag{27}$$

This is an important relationship which tells us that the probability of two particles emitting successively with energies and angular momenta (ε_1, j_1) and (ε_2, j_2) is the same as the probability of a particle with an energy and angular momentum $(\varepsilon_1 + \varepsilon_2, j_1 + j_2)$. And it is easy to see that

$$\begin{aligned}
\Gamma(\varepsilon_1, j_1, \varepsilon_2, j_2, \dots, \varepsilon_i, j_i) &= \Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) \times \dots \times \Gamma(\varepsilon_i, j_i|\varepsilon_1, j_1, \dots, \varepsilon_{i-1}, j_{i-1}) \\
&= \Gamma(\varepsilon_1 + \dots + \varepsilon_i, j_1 + \dots + j_i),
\end{aligned} \tag{28}$$

which is an important relationship we will use later.

The function

$$C(A \cup B; A, B) = \ln \Gamma(A \cup B) - \ln[\Gamma(A)\Gamma(B)] \tag{29}$$

is used to measure the statistical correlation between two events A and B . For the Hawking radiation, the correlation between the two sequential emissions[11–14] can be calculated as

$$\begin{aligned}
\ln \Gamma(\varepsilon_1 + \varepsilon_2, j_1 + j_2) - \ln[\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2)] &= \ln \frac{\Gamma(\varepsilon_1 + \varepsilon_2, j_1 + j_2)}{\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2)} \\
&= \ln \frac{\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)}{\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2)} \\
&= \ln \frac{\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)}{\Gamma(\varepsilon_2, j_2)} \neq 0,
\end{aligned} \tag{30}$$

which shows that the two emissions are statistically dependent, and there are correlations between sequential Hawking radiations of WIH.

The conditional probability $\Gamma(\varepsilon_i, j_i | \varepsilon_1, j_1, \dots, \varepsilon_{i-1}, j_{i-1})$ is the tunneling probability of a particle emitting with energy and angular momentum (ε_i, j_i) after a sequence of radiation from $1 \rightarrow (i-1)$, and conditional entropy taken away by this tunneling particle is then given by

$$S(\varepsilon_i, j_i | \varepsilon_1, j_1, \dots, \varepsilon_{i-1}, j_{i-1}) = -\ln \Gamma(\varepsilon_i, j_i | \varepsilon_1, j_1, \dots, \varepsilon_{i-1}, j_{i-1}). \quad (31)$$

The mutual information for the emission of two particles with energies and angular momenta (ε_1, j_1) and (ε_2, j_2) is defined as[11–13]

$$\begin{aligned} S(\varepsilon_2, j_2 : \varepsilon_1, j_1) &\equiv S(\varepsilon_2, j_2) - S(\varepsilon_2, j_2 | \varepsilon_1, j_1) \\ &= -\ln \Gamma(\varepsilon_2, j_2) + \ln \Gamma(\varepsilon_2, j_2 | \varepsilon_1, j_1) \\ &= \ln \frac{\Gamma(\varepsilon_2, j_2 | \varepsilon_1, j_1)}{\Gamma(\varepsilon_2, j_2)}, \end{aligned} \quad (32)$$

which shows that mutual information is equal to correlation between the sequential emissions, that is to say, information can be carried out by correlations between Hawking radiations.

Let us calculate the entropy carried out by Hawking radiations. The entropy of the first emission particle with an energy and angular momentum ε_1, j_1 is

$$S(\varepsilon_1, j_1) = -\ln \Gamma(\varepsilon_1, j_1). \quad (33)$$

The conditional entropy of the second emission after the first emission is

$$S(\varepsilon_2, j_2 | \varepsilon_1, j_1) = -\ln \Gamma(\varepsilon_2, j_2 | \varepsilon_1, j_1). \quad (34)$$

So the total entropy carried by the two emissions becomes

$$S(\varepsilon_1, j_1, \varepsilon_2, j_2) = S(\varepsilon_1, j_1) + S(\varepsilon_2, j_2 | \varepsilon_1, j_1). \quad (35)$$

Assuming the black hole exhausts after radiating n particles, we have the relationship

$$\sum_i^n \varepsilon_i = E, \sum_i^n j_i = J, \quad (36)$$

where E, J are the mass and angular momentum of the WIH. The entropy carried out by

all the emitting particles is

$$\begin{aligned}
S(\varepsilon_1, j_1, \dots, \varepsilon_n, j_n) &= \sum_{i=1}^n S(\varepsilon_i, j_i | \varepsilon_1, j_1, \dots, \varepsilon_{i-1}, j_{i-1}) \\
&= S(\varepsilon_1, j_1) + S(\varepsilon_2, j_2 | \varepsilon_1, j_1) \\
&\quad + \dots + S(\varepsilon_n, j_n | \varepsilon_1, j_1, \dots, \varepsilon_{n-1}, j_{n-1}) \\
&= -\ln \Gamma(\varepsilon_1, j_1) - \ln \Gamma(\varepsilon_2, j_2 | \varepsilon_1, j_1) - \dots \\
&\quad - \ln \Gamma(\varepsilon_n, j_n | \varepsilon_1, j_1, \dots, \varepsilon_{n-1}, j_{n-1}) \\
&= -\ln[\Gamma(\varepsilon_1, j_1) \times \Gamma(\varepsilon_2, j_2 | \varepsilon_1, j_1) \times \dots \times \Gamma(\varepsilon_n, j_n | \varepsilon_1, j_1, \dots, \varepsilon_{n-1}, j_{n-1})] \\
&= -\ln \Gamma(\varepsilon_1 + \varepsilon_2 + \dots + \varepsilon_n, j_1 + j_2 + \dots + j_n) \\
&= -\ln \Gamma(M, J) = 2\pi(E^2 + \sqrt{E^4 - J^2}) = S_{WIH}, \tag{37}
\end{aligned}$$

where we use the Eq.(28) in the fifth equation. The result shows that the entropy carried out by all the emitting particles is equal to the black hole entropy, so the total entropy is conserved.

We have two comments in the above analysis. Firstly, the energy and angular momentum of emitting particles are not arbitrary, because the black hole should satisfy the cosmic censorship at any time, that is to say, the black hole should satisfy $E^4 \geq J^2$. If the extreme case $E^4 = J^2$ is reached, the radiation will stop because the temperature is zero and the sum of the entropy carried out by Hawking radiation and the remaining entropy of black hole is also conserved. Secondly, Ref.[12] requires that emitting particles have the same angular momentum of unit mass as that of black hole. However we find that the calculation of Parikh and Wilczek's tunneling spectrum does not need this condition. For the s-wave, the particles should radiate along the normal direction of the horizon and the emitting particles' angular velocity should be equal to the angular velocity of black hole Ω , so there is no such constrain on emitting particles' angular momenta.

IV. SUMMARY AND CONCLUSIONS

In this paper we generalize the stationary results to weakly isolated horizon—a dynamical black hole, and find that the nonthermal spectrum of weakly isolated horizon also has correlation, information can be carried out by such correlations, and the entropy is conserved during the radiation process. In our analysis we find that the emitting particle's angular

momentum is very general and is restricted only by keeping the cosmic censorship of black hole.

Acknowledgments

This work is supported by National Natural Science Foundation of China (No. 11275017 and No. 11173028)

-
- [1] S. Hawking, Nature **248**(1974)30.
 - [2] S. Hawking, Commun. Math. Phys. **43**(1975)199.
 - [3] Jacob D. Bekenstein, Phys. Rev. D **7**(1973)2333.
 - [4] J. M. Bardeen, B. Carter, and S. W. Hawking, Comm. Math. Phys. **31**(1973)161.
 - [5] S. W. Hawking, Phys. Rev. D **14**(1976)2460.
 - [6] S. W. Hawking, Phys. Rev. D **72**(2005)084013.
 - [7] C. G. Callan and J. M. Maldacena, Nucl. Phys. B **472**(1996)591.
 - [8] Samir D. Mathur, Class. Quant. Grav. **26**(2009)224001.
 - [9] M. K. Parikh, F.Wilczek, Phys. Rev. Lett. **85**(2000)5042.
 - [10] M. K. Parikh, Int. J. Mod. Phys. D **13**(2004)2351.
 - [11] Baocheng Zhang, Qing-yu Cai, Li You, Ming-sheng Zhan, Physics Letters B, **675**(2009)98.
 - [12] Baocheng Zhang, Qing-yu Cai, Ming-sheng Zhan, Li You, Annals of Physics **326**(2011)350.
 - [13] Baocheng Zhang, Qing-yu Cai, Ming-sheng Zhan, Li You, Int.J.Mod.Phys.D **22**(2013)1341014.
- First Award in the 2013 Awards for Essays on Gravitation.
- [14] M.A.Arzano, J.M.Medved, E.C. Vagenas, J. High Energy Phys. **0509**(2005)037.
 - [15] A. Ashtekar, C. Beetle , S. Fairhurst, Class Quant Grav. **17**(2000)253.
 - [16] A. Ashtekar, C. Beetle, Lewandowski J. Phys. Rev. D **64**(2001)044016.
 - [17] A. Ashtekar, B. Krishnan, Living Rev Rel. **7**(2004)10.
 - [18] Badri Krishnan, Class. Quantum Grav. **29**(2012)205006.
 - [19] Xiaoning Wu, Sijie Gao, Phys .Rev. D **75**(2007)044027.
 - [20] J. Zhang and Z. Zhao, Phys. Lett. B **638**(2006)110.